Fig. 30 (continued –1)

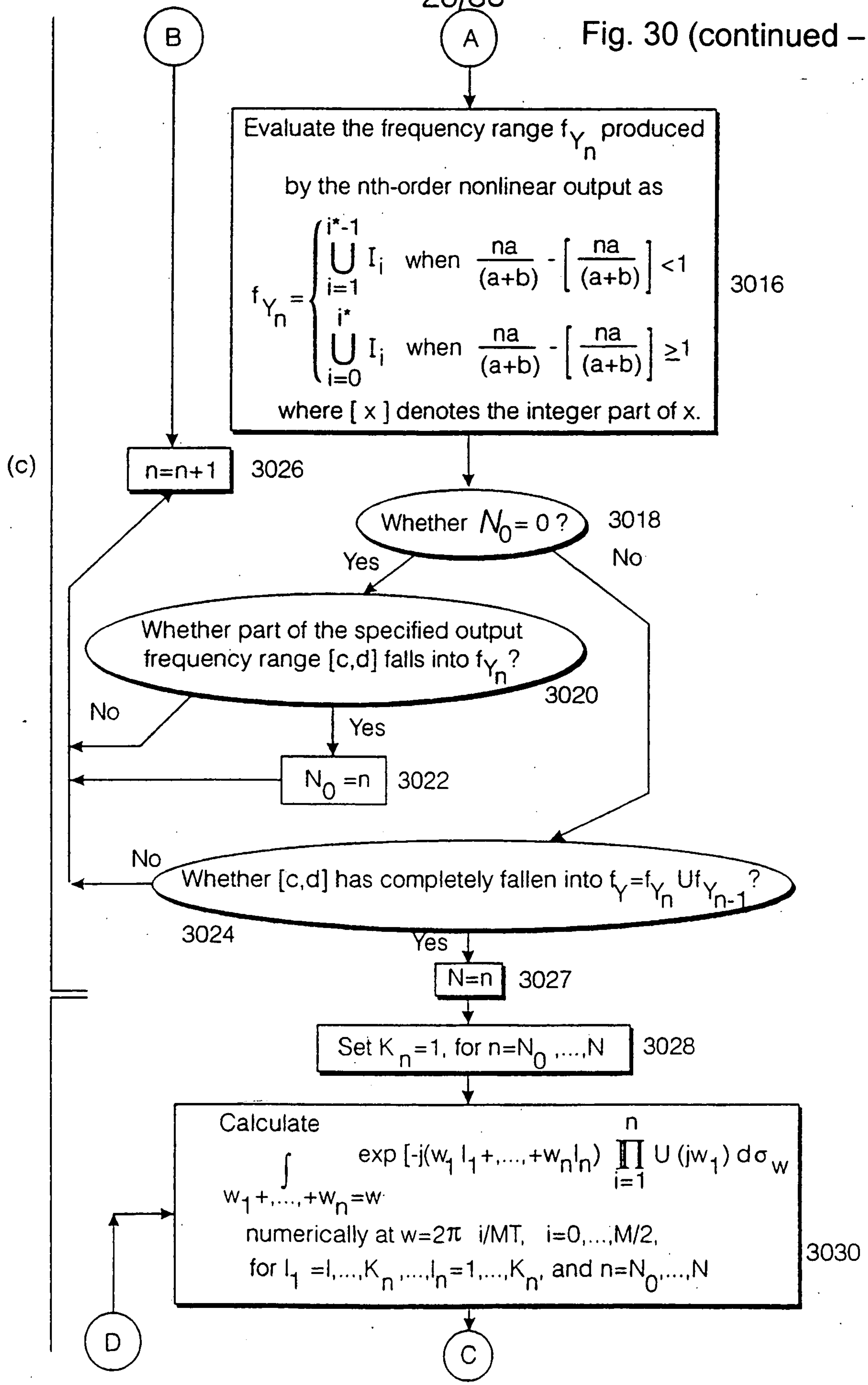

Fig. 30 (continued –2)

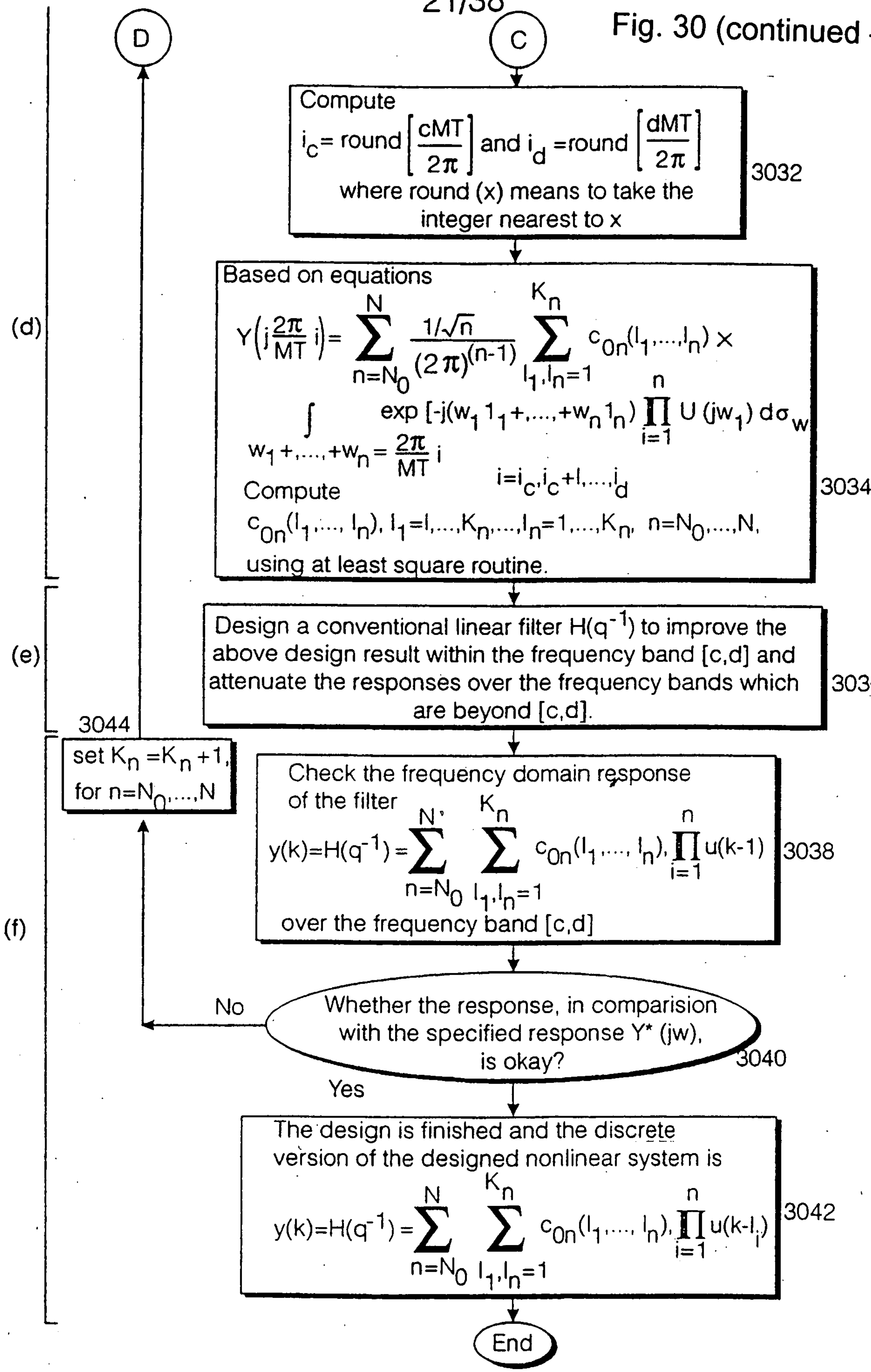

Fig. 31 (continued –1)

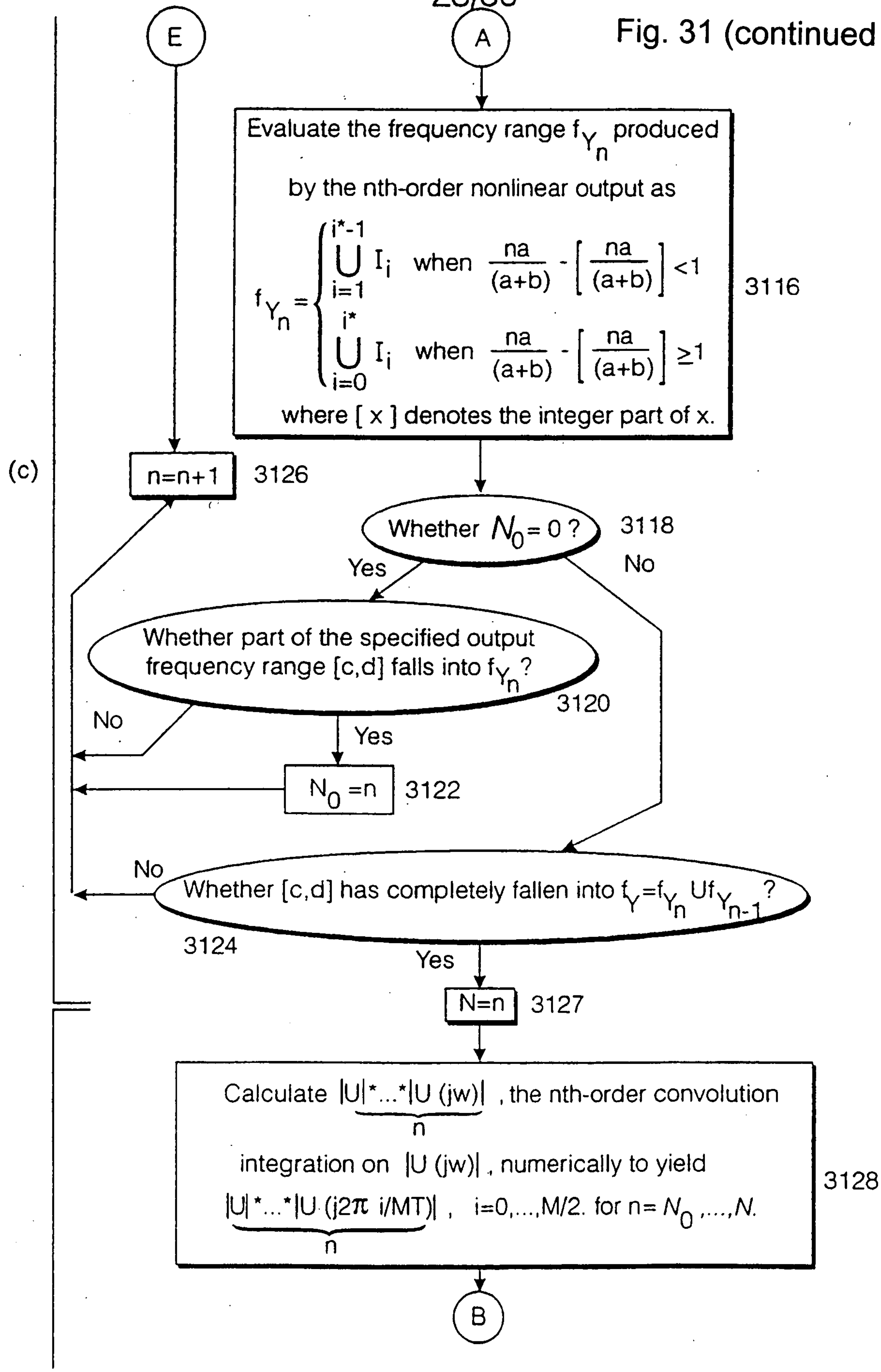

Fig. 31 (continued –2)

B

(d)

Compute
$i_c=\text{round}\left[\frac{cMT}{2\pi}\right]$ and $i_d=\text{round}\left[\frac{dMT}{2\pi}\right]$
where round (x) means to take the integer nearest to x
3132

Based on equations

$$Y^{B*}\left(j\frac{2\pi}{MT}i\right)=\sum_{n=N_0}^{N}\frac{1}{(2\pi)^{(a-1)}}c_n\underbrace{|U|*\cdots*\left|U\left[j\left(\frac{2\pi}{MT}i\right)\right]\right|}_{n}$$

$$i=i_c,i_c+1,\ldots,i_d$$

Compute
$C_n,\ n=N_0,\ldots,N,$
using at least squares routine under the constraint that the results must be positive
3134

(e)

Select NARX model parametres
$K_n,\ c_{0n}(l_1,\ldots,l_n),\ l_1=1,\ldots,K_n,\ldots,l_n=1,\ldots,K_n,$
for $n=N_0,\ldots,N,$ under the constraints

$$C_n=\sum_{l_1=1}^{K_n}\cdots\sum_{l_n=1}^{K_n}\left|c_{0n}(l_1,\ldots,l_n)\right| \qquad n=N_0,\ldots,N$$

and other constraints specified by specific design problems
3136

Check the frequency domain response of the filter

$$y(k)=H(q^{-1})=\sum_{n=N_0}^{N}\sum_{l_1,l_n=1}^{K_n}c_{0n}(l_1,\ldots,l_n)\cdot\prod_{i=1}^{n}u(k-l_i)$$

over the frequency bands beyond [c,d], which should, ideally, be negligible
3138

Whether the frequency response beyond the band [c,d] is ok?
3140

C

D

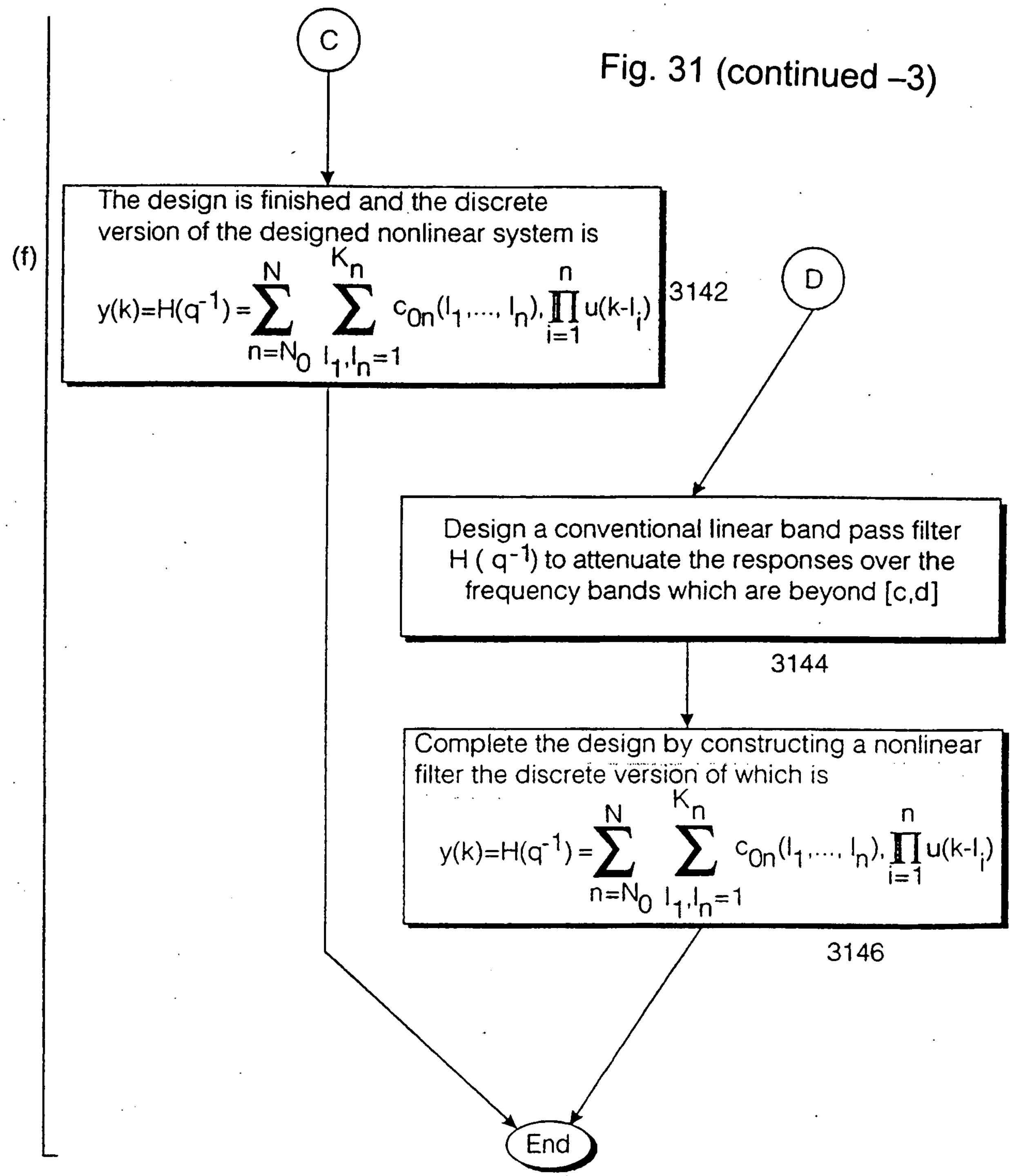

Fig. 31 (continued –3)

where l_b is an integer such that

$$\left|U\left(j\frac{2\pi}{MT}l_b\right)\right| \geq 0.05, \quad \left|U\left(j\frac{2\pi}{MT}l\right)\right| < 0.05 \quad \text{for} \quad l \in \left\{(l_b+1),\cdots,\frac{M}{2}\right\}$$

and l_a is an integer such that

$$\left|U\left(j\frac{2\pi}{MT}l_a\right)\right| \geq 0.05, \quad \left|U\left(j\frac{2\pi}{MT}l\right)\right| < 0.05 \quad \text{for} \quad l \in \{0,\ldots,(l_a-1)\}$$

(4) The relationship between the bound of the output spectrum $Y^B(w)$, the coefficients of the NARX model

$$y(k) = \sum_{n=1}^{N} y_n(k)$$

$$y_n(k) = \begin{cases} \sum_{l_1=1}^{K_n}\cdots\sum_{l_n=1}^{K_n} c_{0n}(l_1,\cdots,l_n)\prod_{i=1}^{n} u(k-l_i) & \text{for } n \geq 2 \\ \sum_{l_1=1}^{K_1} c_{10}(l_1)\, y(k-l_1) + \sum_{l_1=1}^{K_1} c_{01}(l_1)\, u(k-l_1) & \text{for } n = 1 \end{cases}$$

and the spectrum $U(jw)$ is given by

$$Y^B(w) = \frac{1}{\left|1-\sum_{l=l_1}^{K_1} c_{10}(l_1)\exp(-jwl_1)\right|}\sum_{n=N_0}^{N}\frac{1}{(2\pi)^{(n-1)}}\left[\sum_{l_1=1}^{K_n}\cdots\sum_{l_n=1}^{K_n}\left|c_{0n}(l_1,\cdots,l_n)\right|\right]\underbrace{|U|*\cdots*|U(jw)|}_{n}$$

$$= \frac{1}{\left|1-\sum_{l=l_1}^{K_1} c_{10}(l_1)\exp(-jwl_1)\right|}\sum_{n=N_0}^{N}\frac{1}{(2\pi)^{(n-1)}}C_n\underbrace{|U|*\cdots*|U(jw)|}_{n}$$

where

$$C_n = \sum_{l_1=1}^{K_n}\cdots\sum_{l_n=1}^{K_n}\left|c_{0n}(l_1,\cdots,l_n)\right|, \qquad n = N_0,\cdots,N$$

are parameters associated with the NARX model parameters $K_n, c_{0n}(l_1,\cdots,l_n)$, $l_1 = 1,\cdots,K_n,\cdots,l_n = 1,\cdots,K_n$, for $n = N_0,\cdots,N$, $\underbrace{|U|*\cdots*|U(jw)|}_{n}$ denotes the n-dimensional convolution integration for the magnitude $|U(jw)|$ of the spectrum $U(jw)$, which is defined by

$$\underbrace{|U|*\cdots*|U(jw)|}_{n} = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}|U(jw_1)|\cdots|U[j(w-w_1-,\cdots,-w_{n-1})]|dw_1\cdots dw_{n-1}$$

and $N_0 = 1$ when the NARX model involves nonlinear terms from order 1 to N.

Based upon this expression, the structure parameters N and N_0 and the NARX model parameters are determined as below.

(i) Evaluate

$$f_{Y_n} = \begin{cases} \bigcup_{i=1}^{i^*-1} I_i & \text{when } \frac{nb}{(a+b)} - \left[\frac{na}{(a+b)}\right] < 1 \\ \bigcup_{i=0}^{i^*} I_i & \text{when } \frac{nb}{(a+b)} - \left[\frac{na}{(a+b)}\right] \geq 1 \end{cases}$$

where [x] denotes the integer part of x,

$$i^* = \left[\frac{na}{(a+b)}\right] + 1$$

$$I_i = [na - i(a+b), nb - i(a+b)], \quad \text{for } i = 0,\cdots,i^*-1$$

$$I_{i^*} = [0, nb - i^*(a+b)]$$

for n=1,2,... until a value of n is reached such that part of the specified output frequency range $[c,d]$ falls into f_{Y_n}. This value of n is used as the value of N_0.

(ii) Evaluate

$$f_Y = f_{Y_n} \bigcup f_{Y_{n-1}}$$

for n=2,3,... until a value of n is reached such that the frequency range $[c,d]$ falls completely within the corresponding f_Y. This value of n is taken as the value of N.

(iii) Calculate

$$\underbrace{|U|*\cdots*|U(jw)|}_{n}$$

to yield

$$\underbrace{|U|*\cdots*|U(j2\pi\ i/MT)|}_{n}, \qquad \text{for} \qquad i=0,\cdots,M/2$$

using the algorithm

$$\begin{cases} \underbrace{|U|*\cdots*|U(j2\pi\ i/MT)|}_{n} = T\tilde{\tilde{U}}\left[i+\left(\frac{M}{2}-1\right)n\right]\left(\frac{2\pi}{M}\right)^{(n-1)} \quad i=0,\cdots,\frac{M}{2} \\ \left\{\tilde{\tilde{U}}(0),\cdots,\tilde{\tilde{U}}[n(M-1)\right\} = \mathrm{Conv}\left\{\underbrace{[\tilde{U}(0),\cdots,\tilde{U}(M-1)],\cdots,[\tilde{U}(0),\cdots,\tilde{U}(M-1)]}_{n}\right\}, \\ \tilde{U}(i) = \left|U_d\left[j\frac{2\pi}{M}\left(i-\frac{M}{2}+1\right)\right]\right|, \quad i=0,1,\cdots,M-1 \end{cases}$$

for $n=N_0,\cdots,N$, where $\mathrm{Conv}(.)$ denotes the convolution operation and $\tilde{U}(.)$ and $\tilde{\tilde{U}}(.)$ represent the intermediate results of this algorithm.

(iv) Based on the (i_d-i_c+1) equations

$$Y^{B^*}\left(\frac{2\pi}{MT}i\right) = \sum_{n=N_0}^{N}\frac{1}{(2\pi)^{(n-1)}}C_n\underbrace{|U|*\cdots*\left|U\left[j\left(\frac{2\pi i}{MT}\right)\right]\right|}_{n} \quad i=i_c,i_c+1,\cdots,i_d$$

where $i_c=\mathrm{round}\left[\frac{cMT}{2\pi}\right]$, $i_d=\mathrm{round}\left[\frac{dMT}{2\pi}\right]$, and $\mathrm{round}(x)$ means to take the integer nearest to x, use a least squares routine to compute

$$C_n, \qquad n=N_0,\cdots,N,$$

under the constraint that the results must be positive, and then select the NARX model parameters

$$K_n, c_{0n}(l_1,\cdots,l_n), \; l_1=1,\cdots,K_n,\cdots,l_n=1,\cdots,K_n \quad \text{for } n=N_0,\cdots,N$$

under the constraints on the summation of the modulus of the coefficients given by

$$C_n=\sum_{l_1=1}^{K_n}\cdots\sum_{l_n=1}^{K_n}\left|c_{0n}(l_1,\cdots,l_n)\right|, \qquad n=N_0,\cdots,N$$

(v) If necessary, design a classical linear filter, for example, a band pass filter which ideally allows the frequency response to be unity over the frequency

band [c,d] and zero beyond to yield the linear frequency characteristic

$$\frac{1}{\left[1-\sum_{l_1=1}^{K_1} c_{10}(l_1)\exp(-jwl_1)\right]}$$

and therefore determine the parameters associated with K_1, $c_{10}(l_1)$, $l_1 = 1,\cdots,K_1$. Otherwise, all of the parameters of $c_{10}(.)$ can be taken as zero to yield a model having no regression terms associated with the output.

(5) Construct a NARX model as shown in figure (14) using the results obtained in the above (iv) and (v). The nonlinear system illustrated in figure 14 comprises a nonlinear part 1400 and a linear part 1402. It will be appreciated that in figure 14

$$\sum_{l_1,l_n=1}^{K_n} c_{0n}(l_1,\cdots,l_n)\prod_{i=1}^{n} u(k-l_i)=\sum_{l_1=1}^{K_n}\cdots\sum_{l_n=1}^{K_n} c_{0n}(l_1,\cdots,l_n)\prod_{i=1}^{n} u(k-l_i)$$

II.2.2 Three Specific Examples

Example 1

This example illustrates a further implementation or design of a nonlinear system following the above detailed procedure.

(1) The signal to be processed is given by

$$u(t)=2M_u\frac{1}{2\pi}\frac{\sin\alpha t-\sin\beta t}{t}$$

with $\alpha=3.3$, $\beta=1$, and $M_u=1.6$. The frequency spectrum $U(jw)$ of the signal is shown in figure 15 indicating that the real input frequency range is $[1,3.3]$. The requirement for the design is to transfer energy of the original signal to the frequency band $[c,d]=[5.6,7.6]$ with the bound on the output spectrum magnitude specified to be $Y^{B^*}(w)=1.6$ over this frequency band.

(2) Sample u(t) with sampling interval $T=0.01\text{sec}$ to produce

$$u(k)=2M_u\frac{1}{2\pi}\frac{\sin\alpha kT-\sin\beta kT}{kT}=2\times1.6\times\frac{1}{2\pi}\times\frac{\sin 3.3\times0.01k-\sin 0.01k}{0.01k}$$

$$k=-1999,\cdots,0,\cdots,2000$$

and perform a Fast Fourier Transform (FFT) on this series (M=4000) to compute

$$U_d\left(j\frac{2\pi}{M}l\right)=U_d\left(j\frac{2\pi}{4000}l\right)\qquad l=-1999,\cdots,0,\cdots,2000$$

and then to yield

$$U\left(j\frac{2\pi}{MT}l\right)=U\left(j\frac{2\pi}{4000\times0.01}l\right)=0.01U_d\left(j\frac{2\pi}{4000}l\right)\qquad l=-1999,\cdots,0,\cdots,2000$$

the result of which, in the nonnegative frequency range, is shown in figure 16.

Notice the difference between the real spectrum of u(t) in figure 15 and the computed spectrum in figure 16. The differences are due to the errors caused by the FFT operation. The design should and will be performed based upon the computed spectrum to lead to more practical results.

(3) Evaluate the frequency range $[a,b]$ of u(t) from the computed spectrum giving

$$a = 0.6283 \qquad b = 3.7699$$

(4) Design the system structure and parameters

(i) Determination of N_0.

Clearly, the output frequency range contributed by the linear part when the input frequencies are within $[a,b] = [0.6283,\ 3.7699]$ is

$$f_{Y_1} = [a,b] = [0.6283,\ 3.7699]$$

The frequency range f_{Y_2} produced by the second order nonlinearity in this case is obtained as follows.

Since $n = 2$,

$$\frac{nb}{(a+b)} - \left\lfloor \frac{na}{(a+b)} \right\rfloor = \frac{2\times 3.7699}{(3.7699+0.6283)} - \left\lfloor \frac{2\times 0.6283}{(3.7699+0.6283)} \right\rfloor$$

$$= 1.7143 - \lfloor 0.2857 \rfloor = 1.7143 - 0 > 1$$

and

$$i^* = \left\lfloor \frac{na}{(a+b)} \right\rfloor + 1 = \left\lfloor \frac{2\times 0.6283}{3.7699+0.6283} \right\rfloor + 1 = \lfloor 0.2857 \rfloor + 1 = 0 + 1 = 1$$

So

$$f_{Y_2} = \bigcup_{i=0}^{i^*} f_i = \left\{ \bigcup_{i=0}^{i^*-1} [na - i(a+b),\ nb - i(a+b)] \right\} \bigcup [0,\ nb - i^*(a+b)]$$

$$= [na, nb] \bigcup [0,\ nb-(a+b)] = [2\times 0.6283,\ 2\times 3.7699] \bigcup [0,\ 2\times 3.7699 - (0.6283+3.7699)]$$

$$= [1.2566,\ 7.5398] \bigcup [0,\ 3.1416] = [0,\ 7.5398]$$

f_{Y_2} thereby obtained contains part of the specified output frequency range $[c,d] = [5.6,\ 7.6]$. So, N_0 is determined to be $N_0 = 2$.

(ii) Determination of N.

Evaluating $f_Y = f_{Y_n} \bigcup f_{Y_{n-1}}$ for n=2 yields

$$f_Y\big|_{n=2} = f_{Y_2} \bigcup f_{Y_1} = [0,\ 7.5398] \bigcup [0.6283,\ 3.7699] = [0,\ 7.5398]$$

To evaluate $f_Y\big|_{n=3} = f_{Y_3} \bigcup f_{Y_2}$, calculate f_{Y_3} first. In this case

$$\frac{nb}{(a+b)} - \left[\frac{na}{(a+b)}\right] = \frac{3\times 3.7699}{(3.7699+0.6283)} - \left[\frac{3\times 0.6283}{(3.7699+0.6283)}\right]$$
$$=2.5714-[\ 0.4286\ ]=2.5714-0>1$$

$$i^* = \left[\frac{na}{(a+b)}\right]+1 = \left[\frac{3\times 0.6283}{3.7699+0.6283}\right]+1 = [\ 0.4286\ ]+1 = 0+1 = 1$$

So

$$f_{Y_3} = \bigcup_{i=0}^{i^*} I_i = I_0 \bigcup I_1 = [na, nb] \bigcup [0, nb-(a+b)]$$
$$= [3\times 0.6283,\ 3\times 3.7699] \bigcup [0,\ 3\times 3.7699-(0.6283+3.7699)]$$
$$= [1.8848,\ 11.3097] \bigcup [0,\ 6.9115] = [0,\ 11.3097]$$

Therefore

$$f_Y\big|_{n=3} = f_{Y_3} \bigcup f_{Y_2} = [0,\ 7.5398] \bigcup [0,\ 11.3097] = [0,\ 11.3097]$$

$f_Y\big|_{n=3}$ thereby obtained includes the whole specified output frequency range $[5.6,\ 7.6]$. N is therefore determined to be $N=3$.

(iii) Calculate $\underbrace{|U|*\cdots*|U(jw)|}_{n}$ for $n=N_0=2$ and $n=N=3$, respectively to yield

$$\underbrace{|U|*\cdots*|U(j2\pi\ i/MI)|}_{n} = \underbrace{|U|*\cdots*|U(j2\pi\ i/4000\times 0.01)|}_{n} \qquad i=0,\cdots,4000/2,\ n=2 \text{ and } 3$$

The results are shown in figure 17.

(iv) Based on the $(i_d - i_c + 1)$ equations

$$Y^{B^*}(\pi\ i/2000\times 0.01) = 1.6 = \frac{1}{2\pi} C_2\ \ |U|*|U(j\pi i/2000\times 0.01)|$$
$$+\frac{1}{(2\pi)^2} C_3\ \ |U|*|U|*|U(j\pi\ i/2000\times 0.01)|$$
$$i = i_c, i_{c+1}, \cdots, i_{d-1}, i_d$$

with

$$i_c = \mathrm{round}\left[cMT/2\pi\right] = \mathrm{round}\left[5.6 \times 4000 \times 0.01/2 \times \pi\right] = 36$$

$$i_d = \mathrm{round}\left[dMT/2\pi\right] = \mathrm{round}\left[7.6 \times 4000 \times 0.01/2 \times \pi\right] = 48$$

use a least squares routine under the constraint of nonnegative solutions to compute C_2 and C_3, that is, to determine C_2 and C_3, under the constraints of $C_2 \geq 0$ and $C_3 \geq 0$, to minimises the following expression

$$\sum_{i=i_c}^{i_d}\left[1.6-\frac{1}{2\pi}C_2\ |U * |U(j\pi\ i/2000\times 0.01)|-\frac{1}{(2\pi)^2}C_3\ |U * |U * |U(j\pi\ i/2000\times 0.01)|\right]^2$$

The results obtained are

$$C_2 = 0 \qquad \text{and} \qquad C_3 = 3.8367$$

(v) Design, optionally, a linear Butterworth band pass filter to attenuate the frequency components beyond the frequency band $[c, d] = [5.6, 7.6]$ to yield the linear frequency characteristic

$$\frac{10^{-3}\left(0.0986-0.1972q^{-2}+0.0986q^{-4}\right)}{1-3.9633q^{-1}+5.8988q^{-2}-3.9076q^{-3}+0.9721q^{-4}}\Bigg|q = e^{jw}$$

(5) Construct a NARX model as shown in figure 14 with

$$N_0 = 2,\ N = 3,\ K_2 = K_3 = 1,\ c_{02}(1,1) = 0,\ c_{03}(1,1,1) = 3.8367,$$

that is,

$$\sum_{n=N_0}^{N}\sum_{l_1=1}^{K_n}\cdots\sum_{l_n=1}^{K_n} c_{0n}(l_1,\cdots,l_n)\prod_{i=1}^{n}u(k-l_i) = \sum_{l_1=1}^{K_2}\sum_{l_2=1}^{K_2} c_{02}(l_1,l_2)\prod_{i=1}^{2}u(k-l_i) + \sum_{l_1=1}^{K_3}\sum_{l_2=1}^{K_3}\sum_{l_3=1}^{K_3} c_{03}(l_1,l_2,l_3)\prod_{i=1}^{3}u(k-l_i)$$

$$= \sum_{l_1=1}^{1}\sum_{l_2=1}^{1} c_{02}(l_1,l_2)\prod_{i=1}^{2}u(k-l_i) + \sum_{l_1=1}^{1}\sum_{l_2=1}^{1}\sum_{l_3=1}^{1} c_{03}(l_1,l_2,l_3)\prod_{i=1}^{3}u(k-l_i)$$

$$= c_{02}(1,1)u^2(k-1) + c_{03}(1,1,1)u^3(k-1) = 0\times u^2(k-1) + 3.8367u^3(k-1) = 3.8367u^3(k-1)$$

and

$$\left[1-\sum_{l_1=1}^{K_1} c_{10}(l_1)q^{-l_1}\right]^{-1} = \frac{10^{-3}\left(0.0986-0.1972q^{-2}+0.0986q^{-4}\right)}{1-3.9633q^{-1}+5.8988q^{-2}-3.9076q^{-3}+0.9721q^{-4}}$$

which determines the parameters associated with the NARX model parameters K_1 and $c_{10}(l_1)$, $l_1 = 1,\cdots,K_1$.

The output frequency response under the given input is shown in figure 18 indicating that the energy has been transferred to the specified frequency band $[c,\ d] = [5.6,\ 7.6]$ with the magnitude of the response below the specified bound 1.6.

The frequency response of the above design is examined for other input signals below.

In the first case, consider

$$u(t)=\frac{2M_u}{\pi(b-a)t^2}\left[2\cos\frac{(a+b)t}{2}-\cos bt-\cos a\ t\right]$$

where a, b, and M_u are defined as above. The frequency spectrum of the signal is shown in figure 19. The frequency range is clearly the same as that of the signal having the spectrum given in figure 15 and the magnitude of the spectrum also satisfies the condition that

$$|U(jw)|\le M_u=1.6$$

This implies that the frequency response of the designed system to this u(t) should in theory also transfer energy into the frequency band $[c, d]=[5.6, 7.6]$ with the output magnitude frequency response being less than $Y^{B^*}(w)=1.6$ over this frequency band. Figure 20 shows this frequency response and indicates that the actual result is consistent with the theoretical predictions.

For the second case, u(t) was taken as a random process with the frequency spectrum given in figure 21. The frequency spectrum is substantially within the frequency range $[1, 3,3]$ with a magnitude of less than 1.6. Therefore, the same conclusion should apply for the output magnitude frequency response of the designed system to this random input. Figure 22 shows this response and indicates that the energy is transferred to a new frequency band of substantially $[5.6, 7.6]$. Note that the magnitude of the output spectrum over this frequency band is well below the specified bound $Y^{B^*}(w)=1.6$. This is because of the effect of the attenuation which is due to intrakernel interference of the nonlinear mechanism.

The nonlinear filter which has been designed above can be represented by the block diagram shown in figure 23. This

The frequency domain response of the constructed model to the sampled series of the input signal is shown in figure 27. It can be seen that excellent energy attenuation has been achieved by the designed system. It will be appreciated that the input energy in figure 26 has been spread over the designed frequency band by the nonlinear filter.

Example 3

This example shows another application of the present invention to the attenuation of signal energy over unwanted frequency bands using designed nonlinear effects. The example also illustrates the effect of the same design on a different input signal to demonstrate the effectiveness in energy attenuation of the designed system in different circumstances.

(1) Design a nonlinear system to attenuate the energy of the signal

$$u(t) = 2M_u \frac{1}{2\pi} \frac{\sin \alpha t - \sin \beta t}{t}$$

with $\alpha = 3.3$, $\beta = 1$, and $M_u = 1.6$.

(2) The spectrum of the signal is evaluated by sampling the signal with sampling interval $T = 0.01$ sec to produce

$$u(k) = 2M_u \frac{1}{2\pi} \frac{\sin \alpha kT - \sin \beta kT}{kT} = 2 \times 1.6 \times \frac{1}{2\pi} \times \frac{\sin 3.3 \times 0.01k - \sin 0.01k}{0.01k}$$

$$k = -1999, \cdots, 0, \cdots, 2000$$

and then performing a Fast Fourier Transform (FFT) on the obtained time series. The result of the FFT is the same as that shown in figure 16.

The selected NARX model parameters satisfy the relationships

$$\sum_{l_1=1}^{K_1} \left|c_{01}(l_1)\right| = 0 = C_1$$

$$\sum_{l_1=1}^{K_2}\sum_{l_2=1}^{K_2} \left|c_{02}(l_1,l_2)\right| = |c_{02}(1,1)| + |c_{02}(1,2)| = |0.1928| + |-0.1| = 0.2928 = C_2$$

$$\sum_{l_1=1}^{K_3}\sum_{l_2=1}^{K_3}\sum_{l_3=1}^{K_3} \left|c_{03}(l_1,l_2,l_3)\right| = |c_{03}(1,1,1)| + |c_{03}(1,2,2)| = |0.6763| + |-0.3| = 0.9763 = C_3$$

and the different signs selected for

$c_{02}(1,1)$ and $c_{02}(1,2)$

and for

$c_{03}(1,1,1)$ and $c_{03}(1,2,2)$

are in order to give effect to the intra-kernel and inter-kernel interferences, which is to attenuate the energy of the input signal.

The frequency response of the constructed model to the input signal specified above is shown in figure 28 which indicates that the required energy attenuation has been realised.

The frequency response of the design above to the input signal in example 2 gives the result shown in figure 29. The nonlinear system design above clearly also works for the signal in example 2 in energy attenuation although the model was not specially designed for this signal. This is reasonable since the magnitude of the spectrum of the signal in example 2 is less than the magnitude of the spectrum of the signal in this example over almost all of the input frequency band and over the other frequency bands the magnitudes of the spectra of the two signals are all zero. This illustrates that the above design is effective not only for the input based on which the design is implemented but also for other inputs with magnitude frequency characteristics less than the magnitude of the spectrum of the considered input.

III. FLOWCHARTS

Referring to figure 30, there is shown a flow chart 3000 depicting the steps of an embodiment of the present invention in which the output spectrum is specified over a given range of output frequencies.

The signal to be processed and the desired frequency response of the non-linear system to be designed are input via steps 3002 to 3006.

At step 3002, a digital input signal {u(k)} and its sampling interval T are to be provided. The range of output frequencies [c,d] over which the energy of the input signal is to be transformed is given in step 3004. The output frequency range is specified using beginning and end frequencies c and d respectively. The distribution of the energy over the output frequency range [c,d] is requested or specified in step 3006.

The frequency characteristics of the input signal are determined at steps 3008 and 3010. More particularly, the frequency components of the digitised input signal,{u(k)}, are calculated using a Fast Fourier Transform at step 3008. The range of frequency components contained within the input signal is determined from the FFT at step 3010.

Referring to steps 3012 to 3024, the orders of the nonlinearities required to realise a desired nonlinear system and, hence, the energy transformation, are calculated.